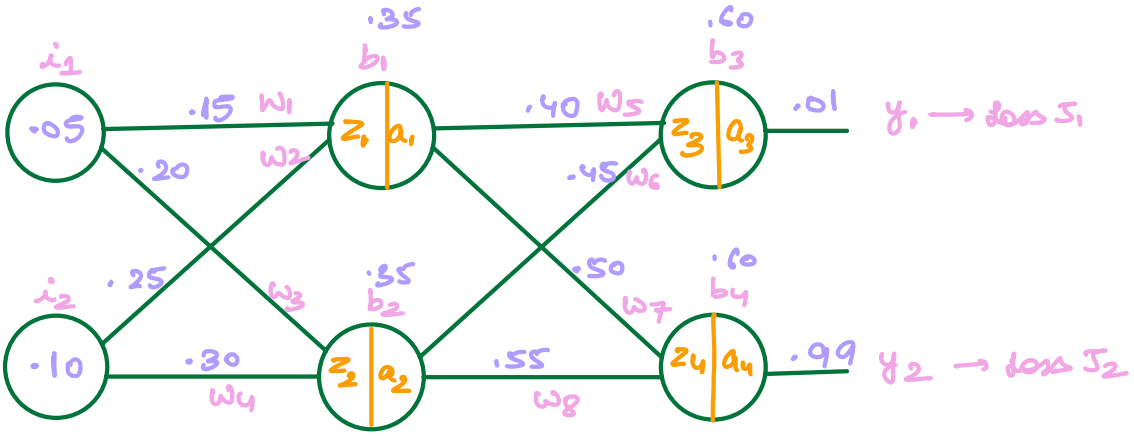
Ques:



loss function: MSE $\frac{1}{m}\sum_{i=1}^{m}(\hat{y}-y)^2$

Activation function: Sigmoid function

$\eta = 0.5$

Forward Pass:

$z_1 = i_1 * w_1 + i_2 * w_2 + b_1 = 0.05 * 0.15 + .10 * .25 + .35 = 0.3825$

$a_1 = \frac{1}{1+e^{-z_1}} = \frac{1}{1+e^{-0.3825}} = 0.593$

$z_2 = i_1 * w_3 + i_2 * w_4 + b_2 = 0.05 * .20 + .10 * .30 + .35 = 0.39$

$a_2 = \frac{1}{1+e^{-z_2}} = 0.597$

$z_3 = a_1 * w_5 + a_2 * w_6 + b_3 = 0.593 * .40 + 0.597 * .45 + .6 = 1.106$

$a_3 = \frac{1}{1+e^{-z_3}} = 0.751$

$z_4 = a_1 * w_7 + a_2 * w_8 + b_4 = 0.593 * .50 + 0.597 * .55 + .6 = 1.225$

$a_4 = \frac{1}{1+e^{-z_4}} = 0.773$

loss : How good w current w are?

$$J = \frac{1}{2m} \sum_{i=1}^{m} (\hat{y} - y)^2$$ $\boxed{m=1}$

$$J_1 = \frac{1}{2}(0.751 - 0.01)^2 = 0.275$$ loss at 1st neuron of output layer

$$J_2 = \frac{1}{2}(0.773 - 0.99)^2 = 0.023$$ loss at 2nd neuron of output layer

$J = J_1 + J_2 = 0.275 + 0.023 = 0.298$ } Total loss

Backpropagation:

→ $\frac{\partial J}{\partial w_5}$?

$$\frac{\partial J}{\partial w_5} = \frac{\partial J}{\partial a_3} \cdot \frac{\partial a_3}{\partial z_3} \cdot \frac{\partial z_3}{\partial w_5}$$

① ② ③

① $\frac{\partial J}{\partial a_3}$

$$J = \frac{1}{2}(a_3 - y_1)^2 + \frac{1}{2}(a_4 - y_2)^2$$

$$\frac{\partial J}{\partial a_3} = \frac{1}{2} \cdot 2(a_3 - y_1) = a_3 - y_1$$

$$= 0.751 - 0.01 = 0.741$$

② $\frac{\partial a_3}{\partial z_3}$

$$a_3 = \frac{1}{1 + e^{-z_3}}$$

$$\frac{\partial a_3}{\partial z_3} = a_3(1 - a_3)$$

$$= 0.751(1 - 0.751) = 0.187$$

③ $\frac{\partial z_3}{\partial w_5}$ = ?

$$z_3 = a_1 * w_5 + a_2 * w_6 + b_3$$

$$\frac{\partial z_3}{\partial w_5} = a_1 : 0.593$$

$$\frac{\partial J}{\partial w_5} : \textcircled{1}\ \textcircled{2}\ \textcircled{3}$$

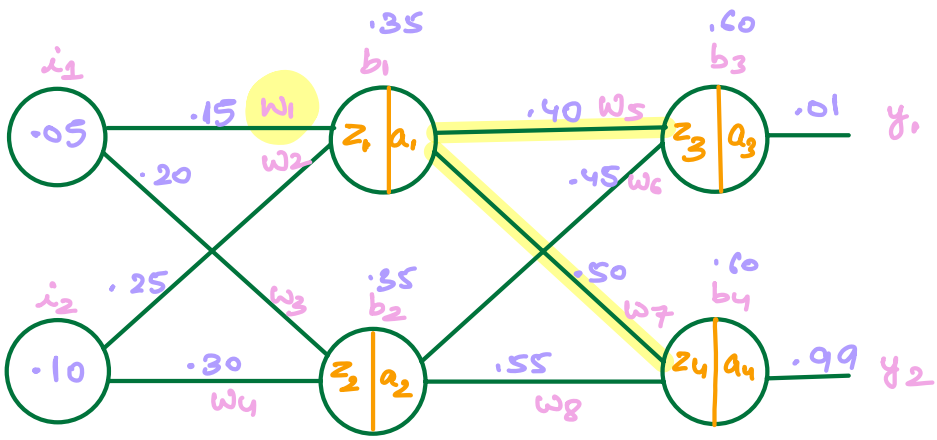
$$= 0.741 * 0.187 * 0.593 : 0.0821$$

$$w_5 = w_5 - \eta \frac{\partial J}{\partial w_5}$$

$$= 0.4 - 0.5 * 0.0821 : 0.359$$

Hidden Layer Neuron

* $\frac{\partial J}{\partial w_1}$?



$$\frac{\partial J}{\partial w_1} = \frac{\partial J}{\partial a_3} \cdot \frac{\partial a_3}{\partial z_3} \cdot \frac{\partial z_3}{\partial a_1} \cdot \frac{\partial a_1}{\partial z_1} \cdot \frac{\partial z_1}{\partial w_1} + \frac{\partial J}{\partial a_4} \cdot \frac{\partial a_4}{\partial z_4} \cdot \frac{\partial z_4}{\partial a_1} \cdot \frac{\partial a_1}{\partial z_1} \cdot \frac{\partial z_1}{\partial w_1}$$

$$= \left(\frac{\partial J}{\partial a_3} \cdot \frac{\partial a_3}{\partial z_3} \cdot \frac{\partial z_3}{\partial a_1} + \frac{\partial J}{\partial a_4} \cdot \frac{\partial a_4}{\partial z_4} \cdot \frac{\partial z_4}{\partial a_1} \right) \cdot \frac{\partial a_1}{\partial z_1} \cdot \frac{\partial z_1}{\partial w_1}$$

$$\textcircled{1}\ \textcircled{2}\ \textcircled{4}\ \ \textcircled{5}\ \textcircled{6}\ \textcircled{7}\ \ \textcircled{8}\ \textcircled{9}$$

④ $\frac{\partial z_3}{\partial a_1}$

$$z_3 = a_1 w_5 + a_2 w_6 + b_3$$

$$\frac{\partial z_3}{\partial a_1} = w_5 = 0.40$$

(5) $\frac{\partial J}{\partial a_4}$

$$J = \frac{1}{2}(a_3 - y_1)^2 + \frac{1}{2}(a_4 - y_2)^2$$

$$\frac{\partial J}{\partial a_4} = \frac{1}{2} \cdot 2(a_4 - y_2) = a_4 - y_2$$

$$= 0.773 - 0.99 = -0.217$$

(6) $\frac{\partial a_4}{\partial z_4}$

$$a_4 = \frac{1}{1+e^{-z_4}}$$

$$\frac{\partial a_4}{\partial z_4} = a_4(1-a_4) = 0.773(1-0.773) = 0.175$$

(7) $\frac{\partial z_4}{\partial a_1}$

$$z_4 = a_1 w_7 + a_2 w_8 + b_4$$

$$\frac{\partial z_4}{\partial a_1} = w_7 = 0.50$$

(8) $\frac{\partial a_1}{\partial z_1}$

$$a_1 = \frac{1}{1+e^{-z_1}}$$

$$\frac{\partial a_1}{\partial z_1} = a_1(1-a_1) = 0.593(1-0.593) = 0.241$$

(9) $\frac{\partial z_1}{\partial w_1}$

$$z_1 = i_1 w_1 + i_2 w_2 + b_1$$

$$\frac{\partial z_1}{\partial w_1} = i_1 = 0.05$$

$$\frac{\partial J}{\partial w_1} = (①②④ + ⑤⑥⑦)⑧⑨$$

$$= (0.741 * 0.187 * 0.40 + (-0.217) * 0.175 * 0.50)\, 0.241 * 0.05$$

$$= (0.0554 - 0.0190)\, 0.0121$$

$$= 0.0364 * 0.0121$$

$$= 0.00044$$

$$w_1 = w_1 - \eta \frac{\partial J}{\partial w_1} = 0.15 - 0.05 * 0.00044 = 0.15 - 0.000022$$

$$= 0.149978$$